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SOME REFINEMENTS TO WING REDUNDANT STRUCTURE ANALYSIS METHODS

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SUMMARY

Some of the trends in the design of wings and control surfaces for high speed vehicles have brought about a need for modifications in the methods of analysis for stress distribution and flexibility. A few characteristics typical of such structures are low aspect ratio, large angle of sweep, thinness, and the use of thick skins, sometimes stiffened by closely spaced integral spars and sometimes forming sandwich structure.

The present report outlines some modifications that can be applied to the matrix force method when the effects of some of these characteristics are important. These refinements do not, of course, affect the application of the method to more conventional structures. It has already proven to be adequate without the refinements for most of the structures handled to date.

The report is concerned with applications of the matrix force method to wing structures idealized as consisting of capstrips carrying axial load only, interconnected by shear panels which can carry shear only. The refinements being introduced are for the purpose of taking into account the Poisson's ratio effect in the case of thick skin wings and for improving upon the handling of the stress coupling effect in the case of non-orthogonal ribs and spars. A system of choosing unit redundant load distributions is also discussed. These refinements have all been treated in some form or another by other authors. However, previous methods of accounting for the Poisson's ratio effect apparently have, except in certain special cases, neglected the fact that while the strain is the same in a spar cap, say, and in the adjacent cover material in the same direction, the stresses may be different because of a state of bi-axial stress in the cover. A way of taking this into account has been included. The question of whether this difference can be significant has not yet been resolved, and it may turn out to be negligible in all practical cases.

These modifications are reviewed in theory, and the formulas and procedures required for practical application are developed and presented in a table. The Poisson's ratio refinement is then applied to the case of a homogeneous laterally loaded flat plate, and excellent agreement with an available plate theory solution is shown.

SYMBOLS

$\alpha\ a_{ij},\ q_i,\ \gamma_{im}$	= symbols defined in ref. 1
$\overline{\sigma}_x, \overline{\sigma}_y, \overline{\sigma}_{xy}$	= stress components referred to rectangular coordinates
E, G	= Young's Modulus, Shear Modulus
ν	= Poisson's Ratio
R_s, R_r, r_s, r_r	= ratios defined where used in the text
t	= skin thickness
$\overline{\sigma}_s, \overline{\sigma}_r, \tau_{sr}$	= stress components referred to oblique coordinates
λ	= sweep angle
b, ℓ	= cell dimensions in rib and spar directions respectively
U	= strain energy
a, h	= side and depth respectively of square plate
M_x, M_y	= bending moments per unit length of sections of plate perpendicular to the x - and y - axes respectively
Δ	= lateral displacement of plate
w	= intensity of a distributed load
$\sigma'_r, \sigma'_s, \tau'_{rs}$	= stress components referred to rectangular coordinates with an axis in the spar direction.

DISCUSSION OF THEORY

Introduction

The report discusses first, for clarity, the Poisson's ratio refinement applied to an unswept wing. Next, a swept wing made up of identical, parallelogram cells is treated and finally the more practical case of a swept wing which has taper and non-parallel ribs and spars. The formulas for the necessary flexibility influence coefficients for this, the most general case, are given in a table. Formulas for the simpler cases can be deduced from these. Although the derivations of the formulas may appear complicated, the formulas themselves and the instructions for using them are summarized compactly in the table.

Idealization - Unswept Case

Consider the problem of analyzing an unswept, thick skin, multi-cell box beam as shown in Figure 1a. Following the conventional matrix force method approach, one lumps the covers into equivalent capstrips over the spars and ribs; the contributions of the ribs and spars themselves are calculated and added in. If appropriate, intermediate shear lag members are introduced as well. Connecting the various spar and rib caps are panels assumed to be capable of carrying shear loads only. Thus, the axial loads in the capstrips vary linearly from one panel point to the next.

Based on this idealization, the required load distributions in the statically determinate structure can be obtained in any of a number of ways. A recommended method is discussed later. The flexibility matrix is calculated from the formulas to be given and the matrix operations are then carried out in the usual manner.

For the derivation of the member flexibilities for a structure of this type, it is convenient to augment the lumped bar and shear panel idealization with a further assumption. It is assumed, for the purpose of accurately expressing the strain energy in terms of the member loads that the stiffening effect of the discrete ribs and spars would not be appreciably altered if they were spread out over the wing covers; in other words, that the stiffened cover can be adequately represented by an orthotropic plate with properly assigned deflectional characteristics. This assumption is employed in the following manner.

Consider a typical rectangular segment of the rib- and spar-stiffened cover, as shown in Figure 1b. The strain energy in such a segment is simply the sum of the skin and stiffener energies. If the strain in

DISCUSSION OF THEORY

Idealization - Unswept Case (Continued)

each direction (spar and rib) is assumed not to vary over the segment, the energy can be written as

$$U = \frac{1}{2E}\left[\sigma_s^{\,2} + \sigma_r^{\,2} - 2\nu\sigma_s\sigma_r + \frac{E}{G}\;\tau_{sr}^{\,2}\right]\ell_s\,\ell_r\,t + a_{spar}\,\ell_s\,\sigma_{spar}^{\,2} + a_{rib}\,\ell_r\,\sigma_{rib}^{\,2}\;, \tag{1}$$

where σ_s, σ_r and τ_{sr} are the skin stresses, (see Fig. 1c), and ℓ_s, ℓ_r, and t are the dimensions of the segment.

It has been taken into account in this expression that in a stiffened cover the stress in the cap is different from the axial stress in the skin in the cap direction. This is because there is a Poisson's ratio effect in the skin but not in the cap.

To simplify the strain energy expression, the symbols r_s and r_r are introduced and defined as the ratios of stiffener area to total cross sectional area of the segment in the spar and rib directions respectively. Also defined are the average stresses $\bar{\sigma}_s$ and $\bar{\sigma}_r$, as the total load divided by the cross sectional area in each of the two directions. Substituting these quantities into (1), the strain energy expression can be manipulated into the form

$$U = \frac{1}{2E}\left[C_{ss}\,\frac{1}{1-r_s}\,\bar{\sigma}_s^{\,2} + C_{rr}\,\frac{1}{1-r_r}\,\bar{\sigma}_r^{\,2} - C_{sr}\cdot 2\nu\,\bar{\sigma}_s\,\bar{\sigma}_r + \frac{E}{G}\;\tau_{sr}^{\,2}\right]\ell_s\,\ell_r\,t \tag{2}$$

where

$$C_{ss} = 1 - \frac{\nu^2\, r_r(1 - r_s)}{1 - \nu^2\, r_s r_r}$$

$$C_{rr} = 1 - \frac{\nu^2\, r_s(1 - r_r)}{1 - \nu^2\, r_s r_r}$$

$$C_{sr} = 1 + \frac{\nu^2\, r_s r_r}{1 - \nu^2\, r_s r_s}$$

DISUCSSION OF THEORY

Idealization - Unswept Case (Continued)

In this expression the C's account for the difference in stress between the skin and stiffeners. If this difference is neglected, each of these quantities reduces to unity. As previously stated, the stiffeners are assumed to be spaced closely enough to permit treating the integrally stiffened cover as an orthotropic plate. With this assumption made, the strain energy of an infinitesimal element of the equivalent orthotropic plate is simply expression (2) with ℓ_s replaced by $d\ell_s$ and ℓ_r replaced by $d\ell_r$. It is now possible to evaluate the member flexibilities by integrating this expression over the entire structure.

The integration is carried out as follows. Referring to Figure 2, the σ_r^2 and σ_s^2 terms are integrated piece-wise over volumes of material that are the same as were lumped into bars and panels previously. For example, in Fig. 2a the integration of the σ_r^2 term is carried out over the cross-hatched area. Fig. 2b shows the area over which the σ_s^2 term is integrated. With the stress varying linearly between panel points, the resulting flexibility influence coefficients are the same as those obtained in ref. 1 except for the "C" factors which take into account the difference in stress between skin and stiffener. The τ_{sr}^2 term integrated over a panel as shown in 2c gives the same expression for shear flexibility as ref. 1. As suggested in ref. 3, the assumption is made that since the contribution of the Poisson's ratio term, $\sigma_s \sigma_r$, to the total strain energy is somewhat smaller than that of the direct stress terms, a less accurate approximation is sufficient for it. Accordingly, in approximating the integral of this term over the structure, it is assumed that the stresses σ_s and σ_r are constant over areas of which the cross-hatched portion of Fig. 2d is typical, and are to be the stress values at the panel point corner. An example would be member loads q_2 and q_3 in Fig. 2e. The resulting flexibility influence coefficient formulas for one cover become

$$\alpha_{1,1} = 2\alpha_{1,2} = \alpha_{2,2} = C_{rr} \cdot b/3a_1E$$

$$\alpha_{3,3} = 2\alpha_{3,4} = \alpha_{4,4} = C_{ss} \cdot \ell/3a_3E \tag{3}$$

$$\alpha_{5,5} = (1/Et)b\ell\,(2 + 2\nu) \text{ or } A/Gt$$

$$\alpha_{2,3} = -\,C_{sr}(1/Et)\nu R_s R_r$$

where the subscripts appended to the α's refer to the member loads shown in Fig. 2e. R_s is the ratio of skin area (cross-hatched side only, Fig.2e)

DISCUSSION OF THEORY

Idealization - Unswept Case (Continued)

to total lumped spar area in a section normal to the spar, and similarly R_r is the ratio of skin area to total lumped rib area in a section normal to the rib. The average skin thickness of the area of integration in each case is t. By employing flexibility influence coefficients of this type with the conventional approach, the Poisson's ratio effect can be adequately taken into account.

It can be shown that in the present case, where there is no sweep, the "C" factors will usually be within a few percent of unity and in no event more than 10% different from unity. Since the Poisson's ratio effect is, itself, responsible for only a small variation in the stress distribution, it can be concluded that consistency will be maintained if these terms are taken equal to unity. This assumption and supporting argument are also made in ref. 8 where a similar problem has been treated.

It may be noted that the member flexibility formulas given in ref. 2 for swept wings reduce to the formulas (2) if there is no sweep and the effect of the C's is neglected.

Idealization - Swept Case

As the next step in the development of the formulas required for the analysis of a practical swept wing structure, consider the case of a swept wing with uniform depth, spars uniformly swept back and identical ribs all parallel to the vehicle centerline. (See Fig. 4a) The infinitesimal element of interest in this case is a parallelogram, and it is convenient to use oblique stresses as shown in Figure 3. These are the same as were employed by Hemp in ref. 7. (The formulas derived in this section have been shown to be in agreement with the work of ref. 7.) Again defining average stresses

$$\bar{\sigma}_s = \frac{q_s}{a_s \sec\lambda} \quad \text{and} \quad \bar{\sigma}_r = \frac{q_r}{a_r \sec\lambda} ,$$

the strain energy expression for the element can be written. This expression and subsequent algebraic manipulation leading to the flexibility formulas is given in Appendix A. The energy expression differs from (2) in two important respects. First there is the introduction of the terms which couple the direct stresses $\bar{\sigma}_s$ and $\bar{\sigma}_r$ with the shear stress τ_{sr}. As pointed out in ref. 3, these are the terms which account for the peaking of the spanwise stress in the covers of swept wings in the vicinity of the rear beam at the wing root. (ref. 1 allows for the $\tau_{sr}\bar{\sigma}_s$ portion of the interaction; the $\tau_{sr}\bar{\sigma}_r$ part is omitted, however. Ref. 1

DISCUSSION OF THEORY

Idealization - Swept Case (Continued)

also accounts for the entire τ_{sr}^2 contribution, but leaves out the Poisson's ratio term $\sigma_s \sigma_r$ altogether.) Secondly, there is the appearance of the parameter μ ($=\nu \cos^2 \lambda - \sin^2 \lambda$) which has replaced Poisson's ratio, ν, in eqn. (2). As a result, unless the sweep angle is small, the "C" factors may become significantly different from unity and may have to be retained in the formulas.

The recommended procedure is to again lump the covers into equivalent capstrips over the ribs and spars with intermediate shear lag members as required. By integrating the strain energy expression for the infinitesimal oblique element over the structure, making similar assumptions as in the unswept case, the member flexibilities can again be determined. The σ_s^2 and σ_r^2 terms yield the same flexibilities as before. The τ_{sr}^2 term is now multiplied by a function of the sweep angle.

In the case of the interaction terms, it is again sufficiently accurate to assume that σ_s and σ_r are constant over areas of which the shaded portion of Fig. 4a is representative, and that they are equal to the values at the corner member loads, as q_2 and q_3 in Fig. 4b. The flexibility influence coefficient formulas for the swept wing with identical parallelogram cells are therefore (referring to Fig. 4b):

$$\alpha_{11} = 2\alpha_{12} = \alpha_{22} = C_{rr} \cdot b/3a_1E$$

$$\alpha_{33} = 2\alpha_{34} = \alpha_{44} = C_{ss} \cdot \ell/3a_3E \tag{4}$$

$$\alpha_{55} = C_{\tau\tau} \cdot (1/Et)b\ell \cos\lambda \, (2 + 2\nu + 4\tan^2\lambda)$$

$$\alpha_{23} = -C_{sr} \cdot (1/Et)R_sR_r \, (\nu - \tan^2\lambda) \cos\lambda$$

$$\alpha_{25} = C_{\tau r} \cdot (1/Et)R_r b \tan\lambda$$

$$\alpha_{35} = C_{\tau s} \cdot (1/Et)R_s \ell \tan\lambda$$

DISCUSSION OF THEORY

Idealization - Swept Case (Continued)

In the preceding R_s is again the ratio of skin area (cross-hatched side only, Fig. 4b) to total lumped spar area in a section normal to the spar, and R_r is again similarly defined. An outline of the derivation of these formulas is given in Appendix A. (These formulas were given, without detailed discussion and with the "C" factors neglected, in ref. 2.)

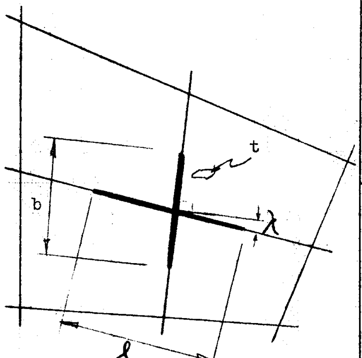
Idealization - Non-Parallel Spars and Non-Parallel Ribs

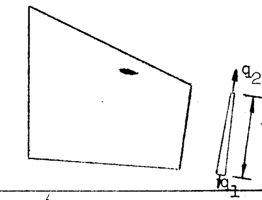
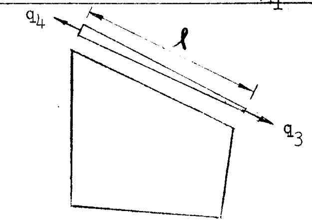
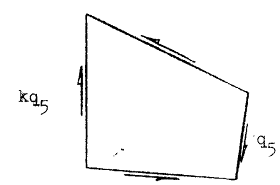
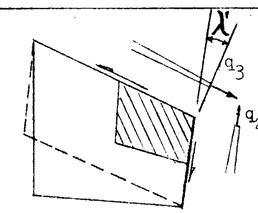
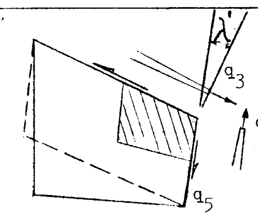
The preceding two sections have discussed cases where the wing exhibited uniformity of stiffness and regularity of planform that would rarely be found in actual practice. The more practical formulas and procedure given in this section, however, can best be developed with the preceding formulas as a basis. In most actual cases, the spars of a wing structure are not parallel; if in addition the wing is swept, some of the ribs are frequently in the free stream direction, while others are normal to the structural axis. In such cases, the σ_s^2 and σ_r^2 strain energy terms are still adequately accounted for by the lumped stringer approach. However, the stringers will generally be tapered and the flexibility expressions will consequently be preceded by the already tabulated factors, "ϕ" (See ref. 1), which take the taper into account. The panels may turn out to be of any general quadrilateral shapes. For these, the τ_{sr}^2 strain energy term can be handled by the expression for quadrilateral shear panels derived as an extension of the work of Garvey (ref. 6), and given in Appendix B. This leaves only the Poisson's ratio and sweep coupling terms unaccounted for.

It is suggested that these terms be handled as follows. Fig. 5 shows a general quadrilateral plate, bounded by spars and ribs. The cross-hatching indicates the area for which the coupling terms are to be approximated. Because of the non-parallelogram shape, the shear flows $k_1 q_5$ and $k_2 q_5$ are unequal. Imagine that the given panel is replaced by the parallelogram panel which is indicated by the dashed lines, and take the shear flow acting on it to be kq_5, where $k = \frac{1}{2}(k_1+k_2)$. The Poisson's ratio terms may now be approximated by the fourth of eqns.(4). The sweep coupling terms may be approximated by the last 2 of eqns. (4), modified for the k factor.

A suggested scheme for calculating averaged values of the C's is given inthe table on the following page, as are the flexibility influence coefficient formulas for a swept wing of arbitrary planform and taper.

- SUMMARY OF FLEXIBILITY INFLUENCE COEFFICIENT FORMULAS -

<table>
<tr><th>Calculation of C's</th><th>Term</th><th>Formula</th><th>Explanation
E - Young's Modulus, G = Shear Modulus</th><th>Illustration</th></tr>
<tr><td rowspan="5">C's are calculated for each panel point based on local values of a_{spar}, a_{rib}, t, b, ℓ and λ; these quantities are averaged where necessary.
$r_s = a_{spar}/(a_{spar} + bt \cos\lambda)$
$r_r = a_{rib}/(a_{rib} + \ell t \cos\lambda)$
$\mu = \nu\cos^2\lambda - \sin^2\lambda$


$C_{ss} = 1 - \frac{\mu^2 r_r(1-r_s)}{1-\mu^2 r_r r_s}$
$C_{rr} = 1 - \frac{\mu^2 r_s(1-r_r)}{1-\mu^2 r_r r_s}$
$C_{sr} = 1 + \frac{\mu^2 r_s r_r + r_r}{1-\mu^2 r_r r_s}$
$C_{\tau\tau} = 1 - \frac{2\mu r_s r_r + r_s + r_r}{1-\mu^2 r_r r_s}$
$C_{\tau r} = 1 + \frac{\mu r_r(1+\mu r_s)}{1-\mu^2 r_r r_s}$
$C_{\tau s} = 1 + \frac{\mu r_s(1+\mu r_r)}{1-\mu^2 r_r r_s}$</td>
<td rowspan="2">Axial Stress</td>
<td>$\alpha_{1,1} = C_{rr_{avg.}} \phi_{11}\, b/3a_1E$
$\alpha_{1,2} = C_{rr_{avg.}} \phi_{12}\, b/6a_1E$
$\alpha_{2,2} = C_{rr_{avg.}} \phi_{22}\, b/3a_1E$</td>
<td>$C_{rr_{avg.}}$ is the average of the C_{rr}'s at q_1 and q_2.
ϕ's are calculated functions of the argument a_1/a_2. See ref. 1.
a_1, a_2 are the idealized cap areas at q_1, q_2.</td>
<td>
</td></tr>
<tr><td>$\alpha_{3,3} = C_{ss_{avg.}} \phi_{33}\, \ell/3a_3E$
$\alpha_{3,4} = C_{ss_{avg.}} \phi_{34}\, \ell/6a_3E$
$\alpha_{4,4} = C_{ss_{avg.}} \phi_{44}\, \ell/3a_3E$</td>
<td>$C_{ss_{avg.}}$ is the average of the C_{ss}'s at q_3 and q_4.
ϕ's are calculated functions of the argument a_3/a_4. See ref. 1.
a_3, a_4 are the idealized cap areas at q_3, q_4.</td>
<td>
</td></tr>
<tr><td>Shear Stress</td>
<td>$\alpha_{55} = C_{\tau\tau_{avg.}} \frac{kA}{Gt} \left\{ 1 + 4\frac{G}{E}\frac{2F}{(a-c)(b-d)(a+b+c+d)} \right\}$</td>
<td>A is the panel area
k is the taper ratio for the side opposite the side where k=1; t is the average thickness
F is a function of a, b, c and d, which are in turn dependent upon the geometry. (See Appx. B)
$C_{\tau\tau_{avg.}}$ is the average of the $C_{\tau\tau}$'s at the four corners of the panel.</td>
<td>
</td></tr>
<tr><td>Poisson's Ratio</td>
<td>$R_r = \frac{1}{2}(1-r_r)$, $R_s = \frac{1}{2}(1-r_s)$
$\alpha_{2,3} = - C_{sr}\, (1/Et)\, R_r R_s \cdot (\nu - \tan^2\lambda')\cos\lambda'$</td>
<td>λ' is the corner sweep angle.
R_s, R_r and C_{sr} are the values of these quantities at the panel corner.</td>
<td>
</td></tr>
<tr><td>Sweep Coupling</td>
<td>$\alpha_{2,5} = C_{\tau r}\, (1/Et)\, \bar{k}\, R_r\, b \tan\lambda'$
$\alpha_{3,5} = C_{\tau s}\, (1/Et)\, \bar{k}\, R_s\, \ell \tan\lambda'$</td>
<td>λ' is the corner sweep angle.
R_s, R_r, $C_{\tau s}$ and $C_{\tau r}$ are the values of these quantities at the panel corner.
$\bar{k}$ is the average of the k's for cross-hatched portion. (See also page 8 and Fig. 5.)</td>
<td>
</td></tr>
</table>

DISCUSSION OF THEORY

Idealization - Non-Parallel Spars and Non-Parallel Ribs (Continued)

The treatment of the coupling terms is admittedly a crude approximation. However, these terms exert a decidedly lesser influence upon structural deflections and stress distributions than do the direct stress terms, and thus the rougher approximation is actually consistent with the rest of the analysis. The trend is indicated in the example given later on. In the case of various practical structures that have been analyzed, a comparison between the results obtained with these coupling terms included and those with the terms omitted altogether, further substantiates this.

After the redundant internal load distribution has been determined using the above method, the skin axial and shear stresses in rectangular coordinates, and the spar and rib stresses, can be calculated by the stress transformation formulas given in Appendix C.

Selection of Redundants and Applied Load Distributions

A matrix formulation practically the same as in ref. 1 has been used by Grzedzielski (ref. 3) and by Argris (ref. 4) in conjunction with a scheme which greatly simplifies and systematizes the determination of the number of redundants and the choice of redundant distributions in multi-cell structures. A discussion of the theory need not be repeated in this report, but the results are essentially that:

1. The number of redundancies (excluding shear-lag members) in a multi-cell structure, with no discontinuities and not including support redundancies, is equal to the number of internal walls, and

2. A simple and systematic set of redundant distributions (again excluding shear lag members) can be formed by applying equal opposite warping groups to each pair of adjacent cells in the structure at their common wall (see Fig. 6).

If shear lag members are present, additional locally self-balancing redundant distributions can be easily devised to handle them. (see, for example, pg. 10, ref. 4)

As pointed out in ref. 4, the applied load distributions, γ_{im} (statically determinate applied load distributions by the customary terminology), may be selected from any distributions that equilibrate the applied load. They should, therefore, be made as simple as possible. (for example: cantilever beams, simply supported beams, distributions according to engineering beam theory, etc.) Strictly speaking, these distributions need not necessarily be statically determinate but are usually selected as such for convenience.

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EXAMPLE

Description

A simple example illustrating the results of the use of some of the discussed techniques is presented in this section. The structure is a square, simply supported solid plate with side a and depth h, subjected to uniform lateral load. This particular example was chosen for two reasons; an exact solution is available for comparison, ref. 5, and it represents the limiting case of a thick skinned, high density core wing. If the recommended semi-monocoque analysis approach works here, it should be even better for the more usual wing structure.

The plate is idealized as a 2 x 2 grid, shown schematically in Fig. 7a, and also as a 4 x 4 and a 6 x 6 grid. In addition to solutions based upon the recommendations of the present report, each case is also solved omitting the Poisson's ratio term. In this way the relative importance of this effect can be qualitatively evaluated.

In determining the equivalent capstrip areas, the familiar technique of maintaining identical bending stiffness is observed; see Fig. 7b. The cover plates are determined in the same manner; thus the equivalent thickness is taken as h/6. The vertical shear webs are assumed to be infinitely stiff; this is in agreement with plate theory assumptions.

The applied loads used to simulate the uniformly distributed loading are concentrated forces acting at the interior grid points. In order to hold down the complexity of the calculations, only unit loads exhibiting double symmetry are considered. Thus it becomes necessary to analyze only one eighth of the plate, giving it, of course, the proper boundary conditions. The numbering system for designating the applied loads is given in Figure 7c.

Results

A comparison of bending moments and deflections at the center of the plate is given in Table 1 for each case investigated. The exact solution of ref. 5 is used as the basis. Bending moment and deflection distributions along the symmetry axes are also presented for the exact solution in Fig. 8, together with the corresponding semi-monocoque solution results. Table 2 gives the calculated flexibility influence coefficients.

An examination of Table 2 indicates that the central deflection due to a unit central load agrees very well with the exact result, the difference varying from 15% for the 2 x 2 grid, to 2% for the 6 x 6.

EXAMPLE

Results (Continued)

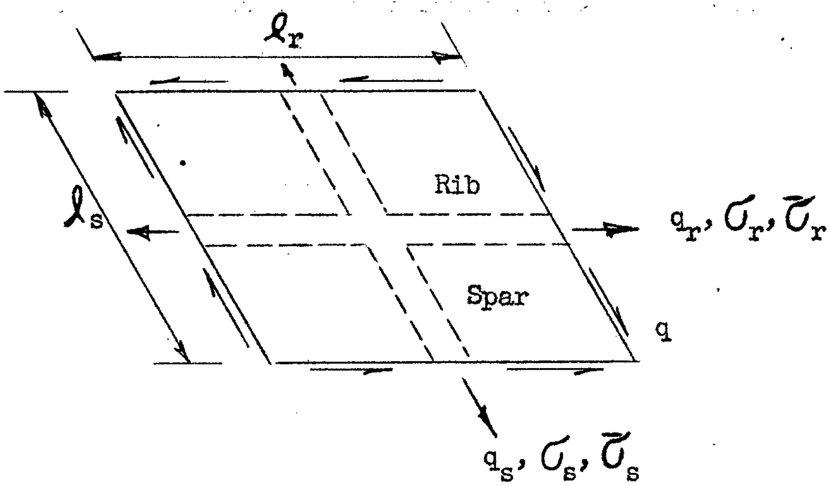
The central deflection due to uniformly distributed load, Table 1, shows a more extreme variation, the difference ranging from 40% for the 2 x 2 to 4% for the 6 x 6. It is felt that the 40% value is the result of the excessively crude representation of the distributed load by a single central force. On the other hand, the central bending moment for the 2 x 2 appears to be artificially close to the exact value. As the number of grid points increases, the representation of the distributed load by concentrated loads should become better and better, and both central deflection and central moment agreement reflect this.

Tables 1 and 2 also indicate that the influence of the Poisson's ratio effect is quite important in this example, being approximately twice as much so in the case of deflections as stresses. The effect should diminish somewhat in the case of wing structures which have concentrations of capstrip material over the webs of the spars and ribs.

In conclusion, it is felt that the example demonstrates that the method of accounting for the Poisson's ratio effect as recommended in this report is quite satisfactory. Furthermore, it would appear reasonable to infer that the recommended method for handling the somewhat similar sweep-coupling effect should work equally well.

APPENDIX A

Strain Energy in a Stiffened Swept Plate



Symbols

ν	= Poisson's ratio
λ	= sweep angle
μ	= $\nu\cos^2\lambda - \sin^2\lambda$
q_s, q_r	= member loads in spar and rib directions, lb.
q	= shear flow, lb/in
t	= plate thickness, in.
a_s	= skin area plus spar area in section normal to spar
a_r	= skin area plus rib area in section normal to rib
E, G	= Young's Modulus, Shear Modulus lb/in^2
R_s	= ratio of skin area to skin area plus spar area in section normal to spar
R_r	= ratio of skin area to skin area plus spar area in section normal to rib
r_s	= ratio of spar area to skin area plus spar area in section normal to spar
r_r	= ratio of rib area to skin area plus rib area in section normal to rib
C_{ss}, C_{rr}, C_{sr}, $C_{\tau\tau}, C_{s\tau}, C_{r\tau}$	= constants in strain energy expression

APPENDIX A

Discussion

The uniform thickness swept plate shown in the illustration is reinforced by an integral spar and rib. The stresses are applied such that the axial strain in the spar direction is uniform throughout plate and spar, and likewise in the rib direction. A shear stress τ is applied to the plate as well. Because of the Poisson's ratio effect, the spar stress is different from the corresponding stress in the plate, and similarly for the rib.

Using the standard stress-strain relations, an expression for the strain energy in the stiffened plate is obtained, as follows:

Strain Energy = Plate Energy + Spar Energy + Rib Energy

$$\text{Strain Energy} = \frac{1}{2E}\Big\{tb\ell\cos\lambda\Big[(\sigma_s^2+\sigma_r^2)\sec^2\lambda - 2\mu\sigma_s\sigma_r\sec^2\lambda + 4\tau_{sr}(\sigma_s+\sigma_r)\tan\lambda\sec\lambda + \tau_{sr}^2\left(\frac{E}{G}+4\tan^2\lambda\right)\Big] + a_{spar}\,\ell\,(\sigma_{spar})^2 + a_{rib}\,b(\sigma_{rib})^2\Big\} \qquad (1)$$

Defining average stresses

$$\bar{\sigma}_s = \frac{q_s}{a_s\sec\lambda}\,,\quad \bar{\sigma}_r = \frac{q_r}{a_r\sec\lambda} \quad \text{and} \quad \tau = q/t$$

and using the definitions of q_s, q_r and q, the following are derived.

$$\sigma_s = \frac{1}{D}\left[\bar{\sigma}_s + \mu r_s\,\bar{\sigma}_r - 2\sin\lambda\, r_s\,(\mu r_s + 1)\tau\right]$$

$$\sigma_r = \frac{1}{D}\left[\mu r_r\,\bar{\sigma}_s + \bar{\sigma}_r - 2\sin\lambda\, r_r\,(\mu r_s + 1)\tau\right] \qquad (3)$$

$$\tau_{sr} = \tau$$

APPENDIX A

Discussion (Continued)

and

$$\sigma_{spar} = \frac{\sec\lambda}{D}\left[(1-\mu^2 r_r)\bar{\sigma}_s - \mu(1-r_s)\bar{\sigma}_r + 2\sin\lambda(1-r_s)(\mu r_r + 1)\tau\right]$$

$$\sigma_{rib} = \frac{\sec\lambda}{D}\left[-\mu(1-r_s)\bar{\sigma}_s + (1-\mu^2 r_s)\bar{\sigma}_r + 2\sin\lambda(1-r_r)(\mu r_s + 1)\tau\right] \qquad (4)$$

where

$$r_s = \frac{a_{spar}}{bt\cos\lambda + a_{spar}}\,, \qquad r_r = \frac{a_{rib}}{\ell t\cos\lambda + a_{rib}}$$

and

$$D = 1 - \mu^2 r_s r_r$$

Substituting expressions (3) and (4) into (1), the strain energy for the segment shown in Fig. 4a can be written as

$$U = \frac{1}{2E}\sec\lambda\Big[C_{ss}\cdot\frac{1}{1-r_s}\bar{\sigma}_s^{\,2} + C_{rr}\cdot\frac{1}{1-r_r}\bar{\sigma}_r^{\,2}$$

$$- C_{sr}\cdot 2\mu\,\sigma_s\sigma_r + C_{\tau\tau}\cdot\left(\frac{E}{G}\cos^2\lambda + 4\sin^2\lambda\right)\tau^2_{sr}$$

$$+ C_{s\tau}\cdot 4\sin\lambda\,\bar{\sigma}_s + C_{r\tau}\cdot 4\sin\lambda\,\bar{\sigma}_r\,\tau\Big]\ell_s\,\ell_r\,t$$

where

$$C_{ss} = 1 - \frac{\mu^2 r_r(1-r_s)}{D} \qquad C_{\tau\tau} = 1 - \frac{2\mu r_s r_r + r_s + r_r}{D}$$

$$C_{rr} = 1 - \frac{\mu^2 r_s(1-r_r)}{D} \qquad C_{s\tau} = 1 + \frac{\mu r_s(1+\mu r_r)}{D}$$

$$C_{sr} = 1 + \frac{\mu^2 r_s r_r}{D} \qquad C_{r\tau} = 1 + \frac{\mu r_r(1+\mu r_s)}{D}$$

APPENDIX B

Flexibility Influence Coefficient for a Quadrilateral Shear Panel From Reference 6

1. General Quadrilateral Panel

A = Area of quadrilateral

t = thickness

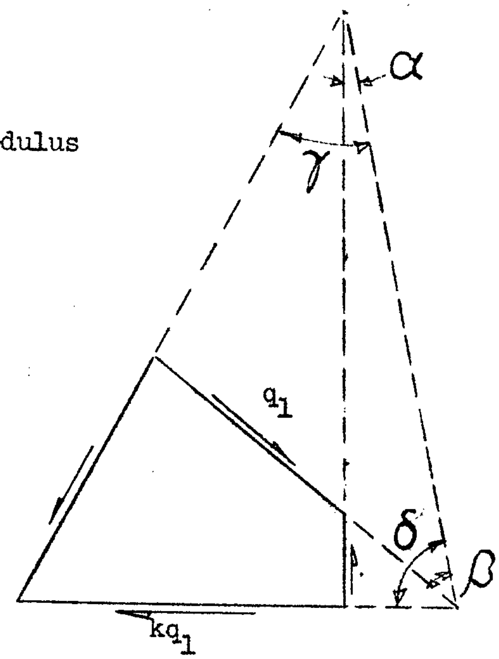
E, G = Young's Modulus, Shear Modulus

a = cot α

b = cot β

c = cot γ

d = cot δ



The flexibility influence coefficient for the general quadrilateral panel shown above is, from ref. 6:

$$\alpha_{all} = k\,\frac{A}{Gt}\left\{1 + 4\,\frac{G}{E}\,\frac{2F}{(a-c)(b-d)(a+b+c+d)}\right\}$$

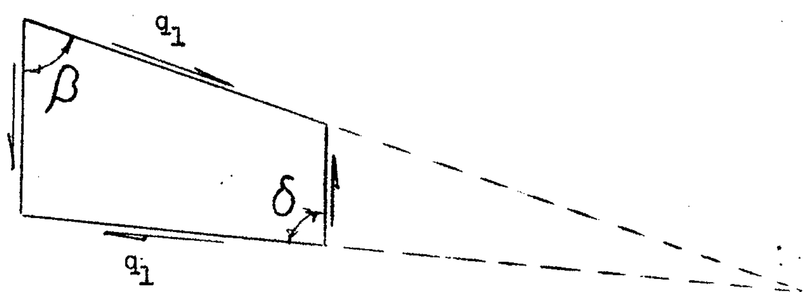
where

$$\begin{aligned} F = {} & (a+b) + \tfrac{2}{3}(a^3+b^3) + \tfrac{1}{5}(a^5+b^5) \quad \log\,(a+b) \\ & + (c+d) + \tfrac{2}{3}(c^3+d^3) + \tfrac{1}{5}(c^5+d^5) \quad \log\,(c+d) \\ & - (b+c) + \tfrac{2}{3}(b^3+c^3) + \tfrac{1}{5}(b^5+c^5) \quad \log\,(b+c) \\ & - (d+a) + \tfrac{2}{3}(d^3+a^3) + \tfrac{1}{5}(d^5+a^5) \quad \log\,(d+a) \\ & + \tfrac{1}{10}(a^2-c^2)(b^3-d^3) + \tfrac{1}{10}(b^2-d^2)(a^3-c^3) - \tfrac{1}{5}(a-c)(b^4-d^4) \\ & - \tfrac{1}{5}(b-d)(a^4-c^4) - \tfrac{2}{3}(a-c)(b-d)(a+b+c+d) \end{aligned}$$

APPENDIX B

Flexibility Influence Coefficient for a Quadrilateral Shear Panel From Reference 6 (Continued)

2. Trapezoidal Panel



If the panel is trapezoidal, the flexibility influence coefficient becomes

$$\alpha_{a11} = \frac{A}{Gt}\left\{1 + 4\,\frac{G}{E}\;\frac{1}{3}(\cot^2\beta + \cot\beta\cot\delta + \cot^2\delta)\right\}$$

APPENDIX C

Calculation of Skin, Spar and Rib Stresses

The use of oblique stress components was recommended to simplify the calculation of member flexibilities in the case of swept wings. For the purpose of designing the wing, the maximum stresses are required. These can be calculated from the idealized structure member loads by the procedure to follow.

To get the skin stress components referred to a rectangular system of coordinates as shown in Fig. 4c, the following formulas (derived from expressions (3) and (4) in Appendix A and the formulas for transformation of stress components from oblique to rectangular coordinates) are applicable.

$$\begin{bmatrix} \sigma'_s \\ \sigma'_r \\ \tau \end{bmatrix} = \frac{1}{1-\mu^2} \left[\begin{array}{c|c|l} 1 + \mu \sin^2\lambda & \mu + \sin^2\lambda & -2 \tan\lambda(\mu+1)(1 \sin^2\lambda)+1 \\ \mu \cos^2\lambda & \cos^2\lambda & -2(\mu+1) \sin\lambda \cos\lambda \\ \sin\lambda \cos\lambda & \sin\lambda \cos\lambda & -2(\mu+1) \sin^2\lambda + 1 \end{array}\right] \begin{bmatrix} q_s/a_s \\ q_r/a_r \\ q/t \end{bmatrix}$$

In the above equation the difference in stress between skin and stiffeners (due to Poisson's ratio) has been neglected. Also, in evaluating the stresses at a point on the wing skin it will be necessary to average the neighboring values of q_s/a_s, q_r/a_r and q/t in some appropriate manner.

To obtain the magnitude and direction of the maximum skin stresses, it is necessary only to apply to the primed stress components the standard formulas for principal stresses in rectangular coordinates.

The spar and rib stresses, under the same assumption, are given by

$$\sigma_{spar} = q_s/a_s \text{ and } \sigma_{rib} = q_r/a_r$$

ILLUSTRATIONS

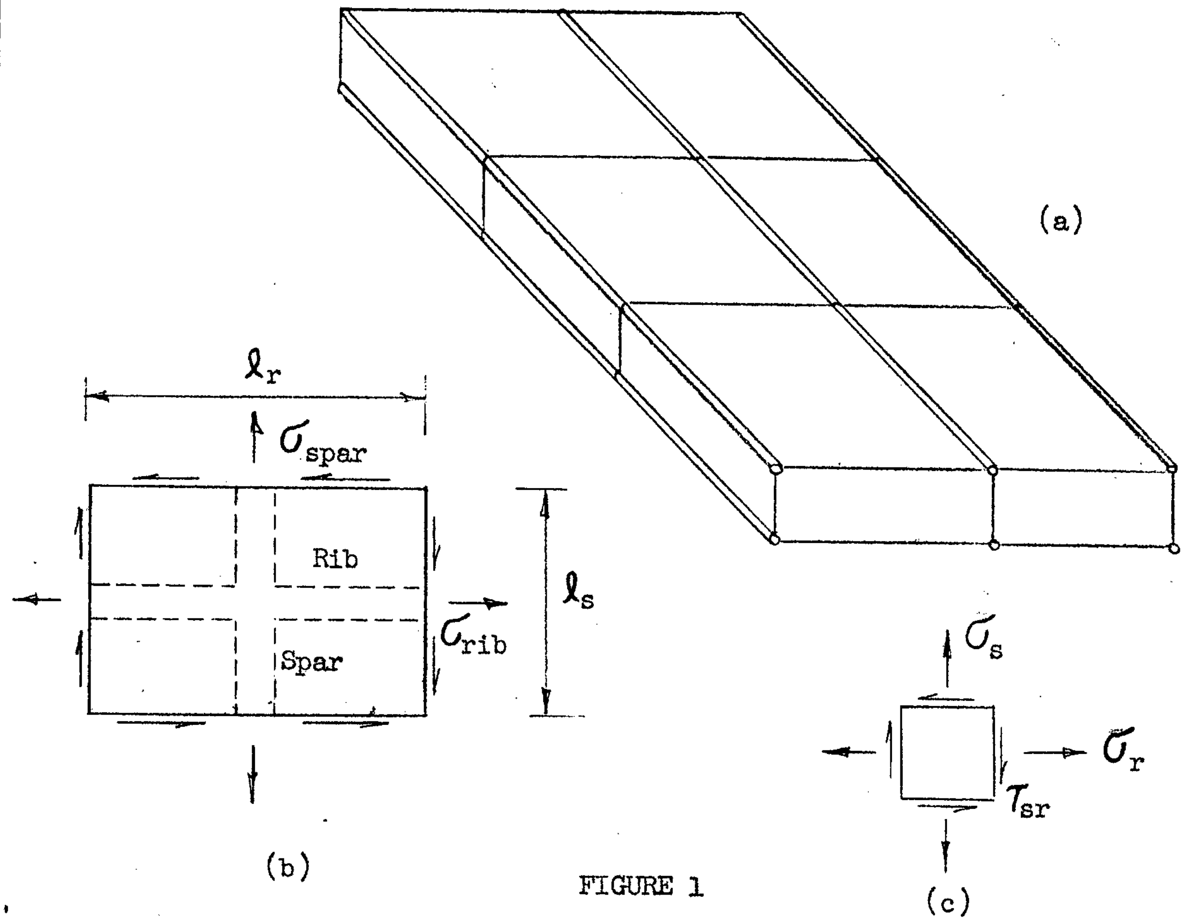


FIGURE 1

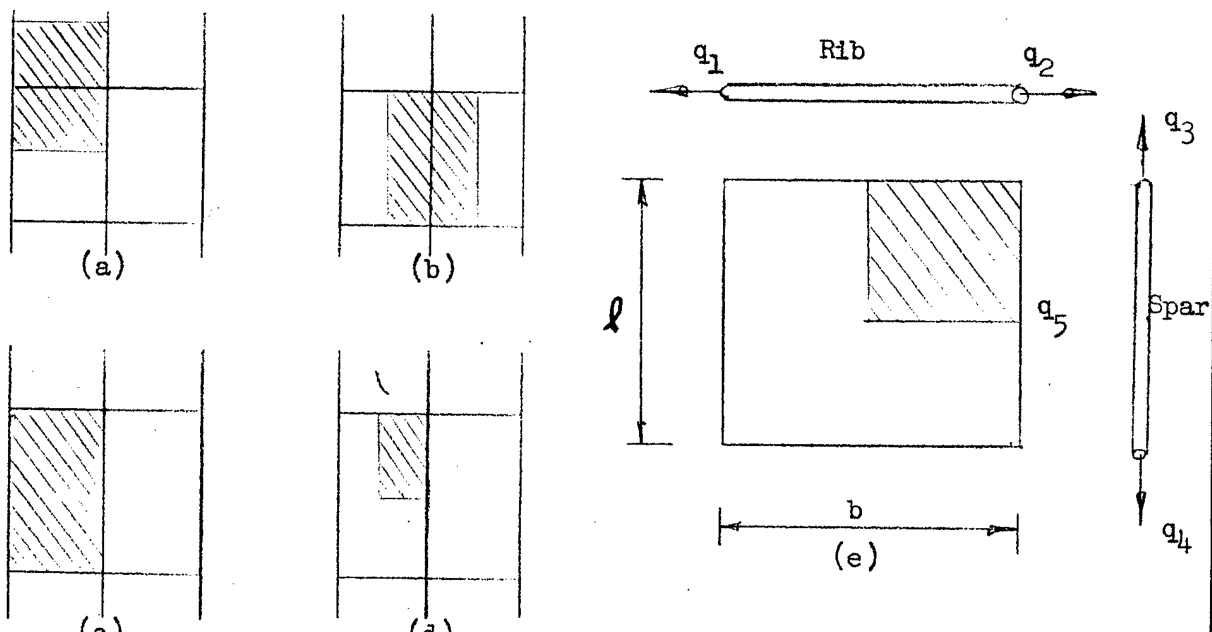


FIGURE 2

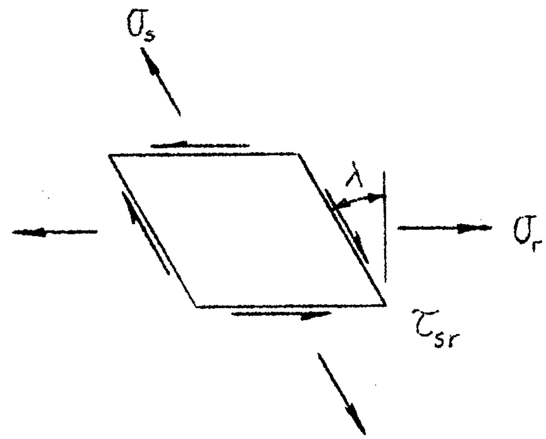


FIGURE 3

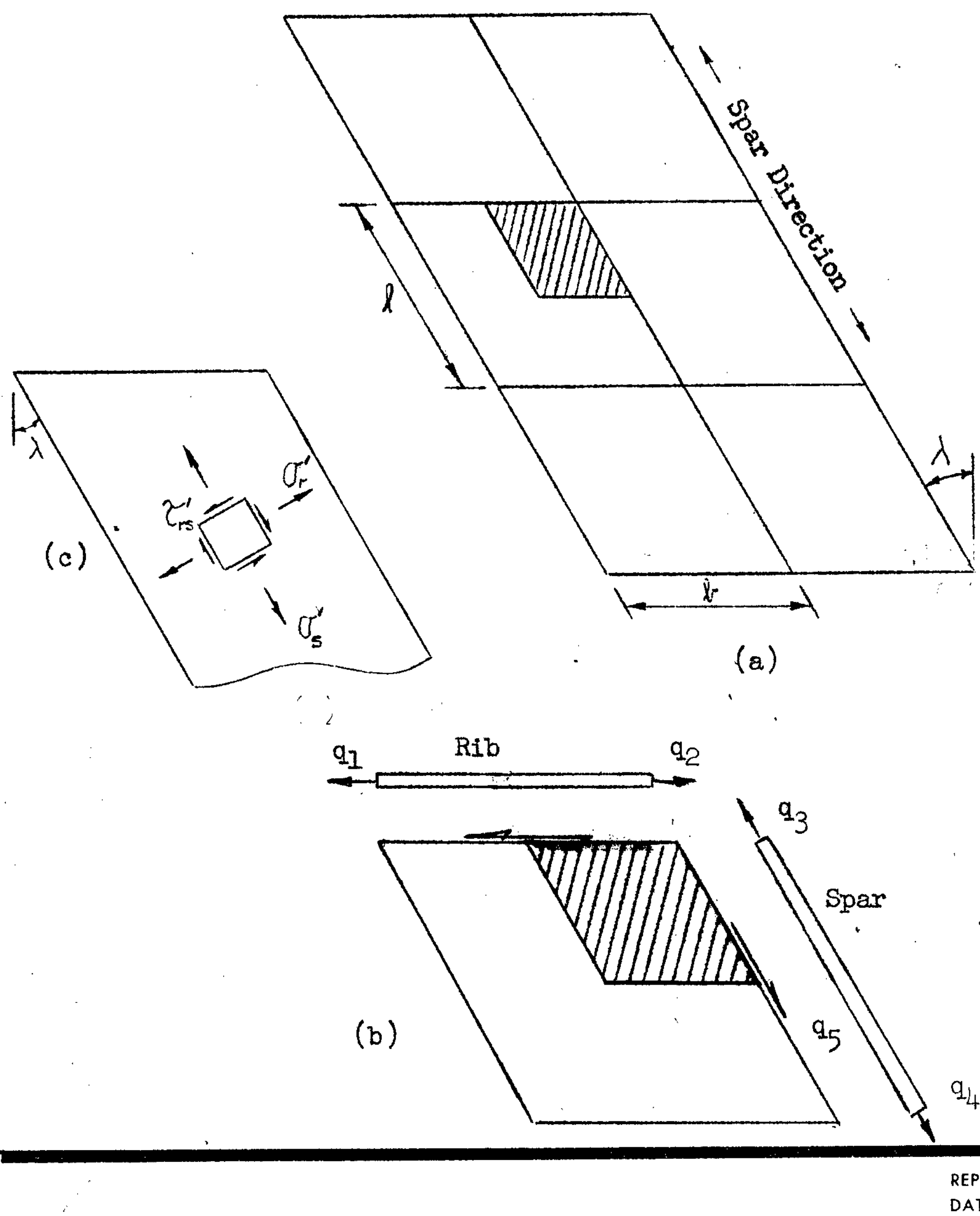


FIGURE 4

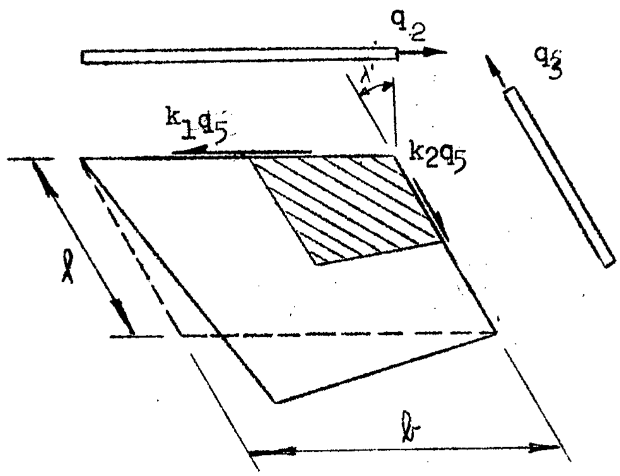


FIGURE 5

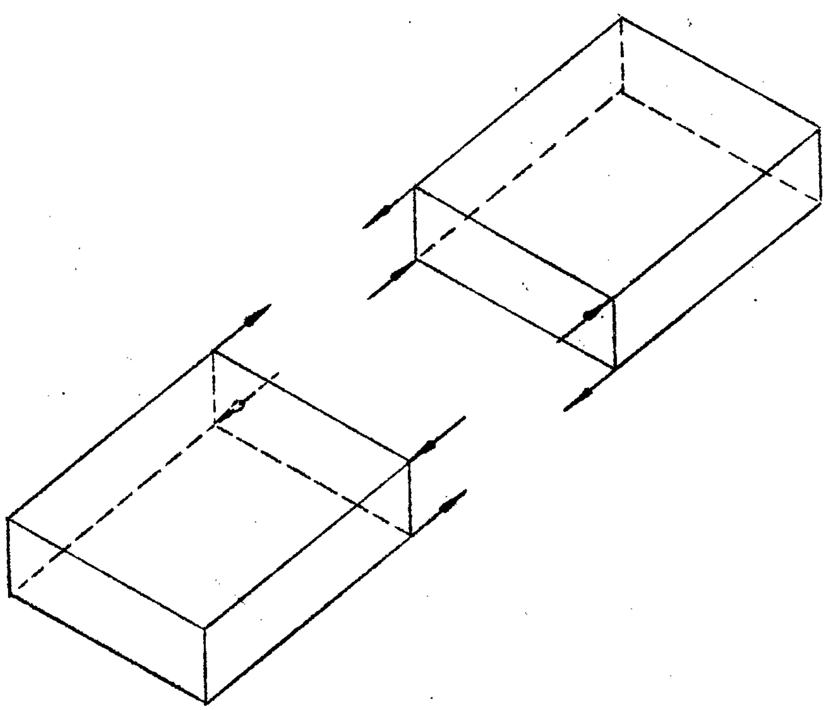


FIGURE 6

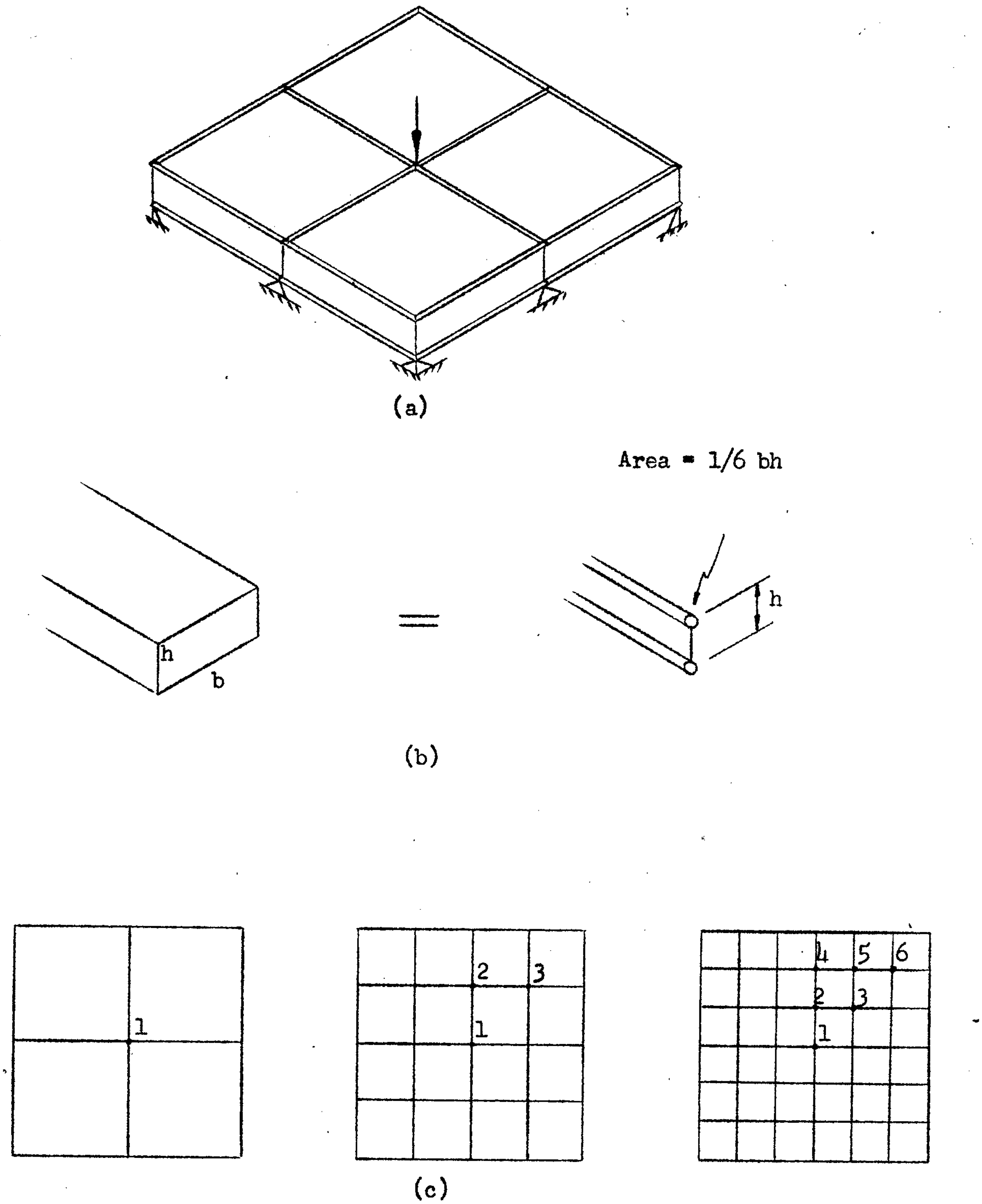


FIGURE 7

BENDING MOMENT AND DEFLECTION AT MIDSECTION OF PLATE UNDER UNIFORM LOADING

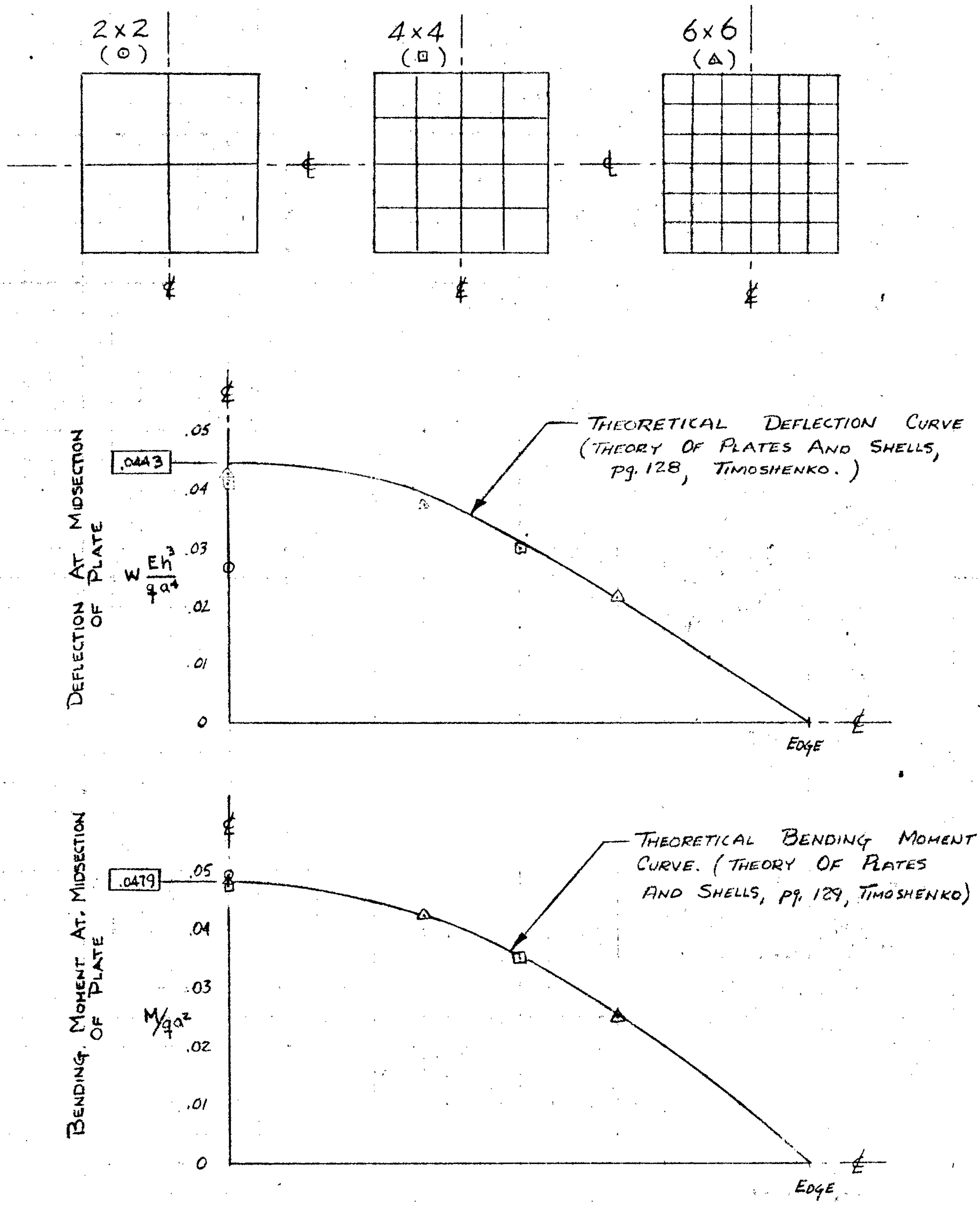


FIGURE 8

TABLE 1

CENTRAL DEFLECTION AND BENDING MOMENT FOR UNIFORMLY LOADED AND SIMPLY SUPPORTED SQUARE PLATE

Grid Dimensions	Poisson's Ratio	No. Member Loads	No. Redundants	No. Terms In Flex. Matrix	No. of Applied Conc. Loads	$\Delta_{max} = \alpha \frac{wa^4}{Eh^3}$ α	$(M_x)_{max} = (M_y)_{max} = \beta wa^2$ β	Means of Solution
exact solution, Ref. 5	0.3					.0443	.0479	
2 x 2	* .3	2	1	2 2	1	.0413(.93) .0268(.60)	.0413(.86) .0487(1.02)	Slide Rule
4 x 4	* .3	7	4	11 14	3	.0522(1.18) .0404(.91)	.0418(.87) .0474(.99)	Desk Calc.
6 x 6	* .3	15	9	27 33	6	.0539(1.22) .0426(.96)	.0419(.87) .0479(1.00)	IBM 650

* Indicates Poisson's ratio term omitted

TABLE 2
FLEXIBILITY INFLUENCE COEFFICIENTS

	1
1	.1265*** .1073** .1651*

2 x 2 Grid Idealization

	1	2	3
1	.1265 .1221 .1603	.0788 .1010	.0523 .0679
2		.0567 .0740	.03941 .05050
3			.03053 .0409

4 x 4 Grid Idealization

	1	2	3	4	5	6
1	.1265 .1246 .1584	.1007 .1267	.0833 .1053	.0532 .0671	.0451 .0572	.0251 .0321
2		.0861 .1090	.0729 .0919	.0477 .0603	.0404 .0511	.0226 .0286
3			.0640 .0811	.0416 .0526	.0364 .0460	.0208 .0263
4				.0304 .0389	.0252 .0317	.0141 .0175
5					.0228 .0290	.0139 .0174
6						.0108 .0141

6 x 6 Grid Idealization

Note: A typical element A_{mn} in the above matrices is the displacement (in non-dimensional form, $\Delta\, Eh^3/a^2$) at a point m in the plate due to a unit load at a point n. The designation of points is given in Figure 7c.

*** Plate theory solution, Ref. 5

** Redundant analysis, Poisson's ratio term included

* Redundant analysis, Poisson's ratio term omitted

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